Topology II
Exam December 21, 2023
Exam time 14.00-17.00

Problems

p1. Let $X = \mathbb{R}/\mathbb{Q}$ be the quoetient space, where $\mathbb{R}$ has the standard Euclidean topology and the quotient is given by equivalence relation $\sim$ satisfying $x \sim y$ if $x = y$ or $x, y \in \mathbb{Q}$. Is the subset $X \setminus \{\mathbb{Q}\}$ of the space X separable in relative topology? Justify your answer.

p2. Let X and Y be topological spaces, $a \in X$, $b \in Y$. Show that

$$C((a,b), X \times Y) \subset C(a, X) \times C(b, Y),$$

where $C(z, Z)$ is the z-component of Z for $z \in Z$. Here $X \times Y$ is the product space with product topology.

p3. Suppose $(X_i)_{i \in I}$ is a family of topological spaces having the property that the product space $X = \prod_{i \in I} X_i$ is locally compact. Show that there exists a finite set $F \subset I$ having the property that X_i is compact for $i \in I \setminus F$.

p4. Let $S = \{0\} \times [-1, 1] \cup \{(x, \sin(1/|x|)) \in \mathbb{R}^2 : x \neq 0\} \subset \mathbb{R}^2$ and $H = \{0\} \times [-1, 1] \cup \{(x, \sin(1/x)) \in \mathbb{R}^2 : x > 0\} \subset S$. Is H a retract of S? Justify your answer.