Topology II
Exam Tuesday October 24, 2023
Exam time 14.00-17.00

Problems

p1. Let $X = \{0\} \cup \mathbb{N}$ and let $\mathcal{T} = \{\emptyset\} \cup \{\{0\} \cup A\colon A \subset \mathbb{N}\}$.

(a) Show that $\mathcal{T}$ is a topology in X.

(b) Show that $\{0\}$ is dense in $(X, \mathcal{T})$.

p2. Let $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ be topological spaces and let $f\colon X \to Y$ be an embedding. Show that the topology $\mathcal{T}$ induced by f from $\mathcal{T}_Y$ is $\mathcal{T}_X$.

p3. Let $\sim$ be the equivalence relation in $\mathbb{R}$ having equivalence classes $\mathbb{Q}$ and $\mathbb{R} \setminus \mathbb{Q}$, that is, $x \sim y$ if either $\{x, y\} \subset \mathbb{Q}$ or $\{x, y\} \subset \mathbb{R} \setminus \mathbb{Q}$. Show that every continuous map $f\colon \mathbb{R}/{\sim} \to \mathbb{R}$ is constant, where $\mathbb{R}/{\sim}$ has the quotient topology.

p4. Let $(X, \mathcal{T}_X)$ be a topological space, where $X = \{0, 1\}$ and $\mathcal{T}_X = \{\emptyset, \{0\}, X\}$. For which $j = 1, \ldots, 4$ the product space $X \times X$ is a T_j-space? Justify your answer.