

University of Helsinki  
Master's Programme in Mathematics and Statistics  
Mathematical logic, spring 2019  
2nd mid-term exam  
3.5.2019

1. Let  $T$  be an  $\{E\}$ -theory such that for every  $n \in \mathbb{N}$  there exists a model  $\mathcal{M}_n \models T$  in which  $E$  defines an equivalence relation with  $n$  equivalence classes. Show that  $T$  has a model where  $E$  defines an equivalence relation with infinitely many equivalence classes.
2. Show that the function  $F : \mathbb{N} \rightarrow \mathbb{N}$  that produces the Fibonacci sequence ( $F(0) = 0$ ,  $F(1) = 1$ , and for  $n > 1$ ,  $F(n) = F(n-1) + F(n-2)$ ) is primitive recursive.
3. Show that there is no recursive function  $f : \mathbb{N}^2 \rightarrow \mathbb{N}$  with the property that if  $g : \mathbb{N} \rightarrow \mathbb{N}$  is a recursive function, then there is some  $n \in \mathbb{N}$  such that for all  $m \in \mathbb{N}$  one has

$$f(n, m) = g(m).$$

4. State and prove Gödel's first incompleteness theorem. You may use earlier results from the course.