University of Helsinki
Master's Programme in Mathematics and Statistics
Mathematical logic, spring 2019
1st mid-term exam
5.3.2019

1. Show directly from the definition of provability that

$$\{(A \to (C \to B)), (\neg C \to \neg A)\} \vdash (A \to B).$$

2. Show that

$$\forall ((\exists v_0 P v_0 \wedge \exists v_0 Q v_0) \to \exists v_0 (P v_0 \wedge Q v_0)).$$

3. Let $L = \{R\}$ be a vocabulary with a binary relation symbol R. Consider the L-structure $(\mathbb{Z}, <)$, where $<$ is the natural order on $\mathbb{Z}$. Is the relation $S = \{(a, a+1) : a \in 2\mathbb{Z}\}$ definable, where $2\mathbb{Z}$ is the set of even integers? Justify your answer.

4. (a) Show that if L is a vocabulary, φ and ψ are logically equivalent L-formulas (i.e., $\varphi \leftrightarrow \psi$ is valid), and t is an L-term such that both $FVF(t, v_0, \varphi)$ and $FVF(t, v_0, \psi)$ hold, then also $\varphi(t/v_0)$ and $\psi(t/v_0)$ are logically equivalent.

   (b) Give an example showing that the claim is not true without the FVF-assumptions.