

Exam: Introduction to bifurcation theory (I)
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You only choose five questions in the following, all questions carry equal weight. Success!

Question 1. Are the following systems diffeomorphic?

$$\begin{cases} \dot{x}_1 = -x_1, \\ \dot{x}_2 = -x_2, \end{cases} \quad \text{and} \quad \begin{cases} \dot{x}_1 = -x_1, \\ \dot{x}_2 = -2x_2. \end{cases}$$

Question 2. If the solution $x(t)$ of equation $\dot{x} = f(x)$ (here $f \in C(\mathbf{R}^n)$) has limit x^* , i.e., $x(t) \rightarrow x^*$ (as $t \rightarrow +\infty$), then $f(x^*) = 0$, i.e., x^* is a fixed point.

Question 3. Consider $\begin{cases} \dot{x} = -xy - 3x^7, \\ \dot{y} = -y + x^2. \end{cases}$ Determine the stability of the equilibrium at the origin (using the center manifold theorem).

Question 4. What are the definitions of the following

- (a) two systems are topologically equivalent;
- (b) two systems are smoothly equivalent (or diffeomorphic);
- (c) two systems are orbitally equivalent.

What are the relations between (a), (b) and (c).

Question 5. What are the definitions of bifurcation and bifurcation diagram for a system? Draw the bifurcation diagram for the following system:

$$\dot{x} = f(x, u) = ux - x^3, \quad x \in \mathbf{R}^1,$$

$$u \in \mathbf{R}^1.$$

Question 6. Consider $\begin{cases} \dot{x} = u + x^2 + y^2, \\ \dot{y} = -y + x^2, \end{cases} \quad u \in \mathbf{R}^1.$

What bifurcations this system undergoes at an equilibrium at the origin when u passes zero?