

University of Helsinki  
Bachelor's programme in mathematical sciences  
MAT21030 Elements of set theory I  
Course exam  
4.3.2024

**No calculators, charts or other extra material allowed.**

*Leave a few empty rows for grading notes at the top of the first page of your answer sheet.*

- Simplify  $(\bigcup A) \cap (\bigcup B)$ , where  $A = \{\emptyset, \{\emptyset, \{\emptyset\}\}$  and  $B = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}, \{\{\emptyset\}\}\}$ .
  - Prove or disprove (i.e., provide a proof or a counter example): For any sets  $A$  and  $B$

$$\mathcal{P}A \cup \mathcal{P}B = \mathcal{P}(A \cup B)$$

- Show that every natural number is a transitive set.
- Show that addition of reals is commutative, i.e.,

$$x +_{\mathbb{R}} y = y +_{\mathbb{R}} x$$

for all  $x, y \in \mathbb{R}$ . State without proof the results from earlier levels of the construction that you use in your proof.

- Show that for any  $n \in \omega$ , any proper subset of  $n$  is equinumerous to some  $m$  less than  $n$ . (Hint: induction)