Department of Mathematics and Statistics
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Exam on
COMPLEX ANALYSIS I
6.5.2019

Only pencils, erasers, and rulers are allowed.
No electronic devices or formula booklets are permitted.
You have 3 hours to complete the exam.

0.1. **Problem.** Determine whether the following functions are analytic.
(a)
$$f(z) = z \operatorname{Re} z.$$
(b)
$$g(z) = \frac{z}{\bar{z} - 2019}.$$

0.2. **Problem.** Let f be an analytic function in the whole complex plane. If there exists $M > 0$ such that $|f(z)| > M$ for all $z \in \mathbb{C}$, show that f is constant in the whole complex plane.

0.3. **Problem.** Let $z_1, z_2 \in \mathbb{C}$ and $r > 0$ be given such that
$$|z_1| < r < |z_2|.$$
If $\sigma = 5\gamma$ is a cycle where $\gamma : [0, 2\pi] \to \mathbb{C}, \gamma(t) = r\exp(it)$, evaluate the integral
$$\int_\sigma \frac{dz}{(z - z_1)(z - z_2)}.$$

0.4. **Problem.** Show that
$$\int_0^\infty \frac{1 - \cos x}{x^2}\, dx = \frac{\pi}{2}.$$