Exam: Combinatorics Spring 2018

Date: 3.5.2018

Remember to write your name and student number on the answer sheet.

1. (20 points) How many pairs (X, Y) exists such that $X, Y \subseteq \{1, \ldots, n\}$ and X and Y are disjoint?

2. (20 points) Show that if $n+1$ distinct integers are chosen from the set $\{1, 2, \ldots, (k+1)n\}$ then there are always two which differ by at most k.

3. (20 points) Prove the following identity: For all non-negative integer n,

$$\sum_{k=0}^{n} \binom{2k}{k} \binom{2n-2k}{n-k} = 4^n.$$

Catalan?

4. (20 points) A circle with center o is the set of all points in the plane that are at a fix given distance, r, from o. Let $x_1, x_2, \ldots, x_{2n}$ be $2n$ points on a line $\mathcal{L}$. In how many ways can n circles be drawn such that for every circle $\mathcal{O}$, there are $1 \leq i, j \leq 2n$ such that $x_i\bar{x}_j$ is a diameter of $\mathcal{O}$ and doesn't intersect with any of the other circles.

5. (20 points) We denote by S_n the set of permutations of $\{1, 2, \ldots, n\}$. How many permutations $\pi \in S_n$ exists such that $\pi(i) - \pi(i+1) \leq 1$ holds for all $1 \leq i < n$?

Stirling ↑