Advanced Calculus MAT11008

Course Examination

March 4, 2019

Not any kind of notes are allowed in the exam. A calculator is allowed.

Solve each problem. Justify your answers by presenting steps of reasoning or computations as well as justifications for using known rules and results when needed.

1. Use the Triangle Inequality to show that
$$|a-b| \geq \big||a|-|b|\big|.$$

2. Let $S = \left\{\frac{\sqrt{x}}{x+1} : x > 0\right\}$. Show that $\sup S = \frac{1}{2}$ and $\inf S = 0$.

3. Let $a_0 \geq 3$ and $a_{n+1} = 1 + \sqrt{1+a_n}$ for $n \in \mathbb{N}$.
   a. Show by induction that $a_n \geq 3$ for all $n \in \mathbb{N}$.
   b. Show that the sequence (a_n) is decreasing.
   c. Find the limit of the sequence.

4. Assume the function f is defined on a closed interval I and furthermore there is a number M so that
$$|f(x) - f(x_0)| \leq M|x - x_0|$$
whenever $x, x_0 \in I$.

   Show that f is uniformly continuous.