

Physics of Biological Physics (MATR332)

University of Helsinki, Spring 2024

Examination: March 08, 2024 (5 exercises; maximum number of points is 30)

As to background information that is allowed during the examination:

- Calculator is ok

Questions are in English but you can write down your answers in English or Finnish.

Assignment 1 [max. 9 points]

Please consider the below statements and conclude whether they are true, or not. Further, *justify your reasoning*.

- In a system under consideration, there is salt dissolved in a water liquid, interacting with a solid surface. The sodium ions of the salt are adsorbed (attached) to the charged surface. The temperature of the system is low, around 10 degrees Celsius. Now, if the system temperature is allowed to increase to reach room temperature (around 20 degrees Celsius), then changes in this system are capable of producing mechanical work.
- Consider a fish swimming in a lake. The size of the fish suddenly increases by 50%, and at the same time the velocity of the fish decreases by 50%. Despite these changes, the Reynolds number characterizing the motion of the fish remains constant.
- You are working on your MSc thesis project. One of the tasks you need to do is to separate selected molecules (type A) from the other molecules (type B, C, ...) in this solution. To do this, you use sedimentation, and it works very well.
- Consider molecules found on planet X in liquid nitrogen at a temperature of +73 K. Assuming that the forces between the molecules under these conditions on planet X are similar to forces in liquid water at +37 degrees Celsius on planet Earth, then this implies that on Earth these molecules would be almost 10 times bigger than on planet X.
- If you planned to use the equation of state of the ideal gas model to describe the behavior of a weakly interacting system, when the system temperature approaches zero Kelvin, your colleague would say that "no chance, never going to work out". Your colleague would be right.
- pH outside a cell is 7.5. Inside the cell it is 6.5. You consider a water-soluble protein in which there is the alpha-COOH group. This group has $pK_a = 2.1$. When the protein is translocated from the extracellular space to the cell's cytosol (from outside to the inside), the alpha-COOH group becomes more protonated.

Assignment 2 [max. 6 points]

Diffusion of particles is described by the diffusion length $L_D = (2dDt)^{1/2}$, where d is dimensionality (e.g., $d = 2$ for 2D motion along a surface), D is the diffusion coefficient, and t is time. The diffusion length then describes how far from the origin the particle moves (on average) during a time scale t if the diffusion coefficient has a value of D . Now assume that the diffusion coefficient of a membrane protein is $D = 10^{-12} \text{ cm}^2/\text{s}$ and the protein diffuses in 2D along the plasma membrane that surrounds a cell, whose radius is R .

- Now we want the protein to diffuse (like a random walker) to the other side of the cell in 60 seconds. Under these circumstances, what should be the radius R of the cell? [Hint:

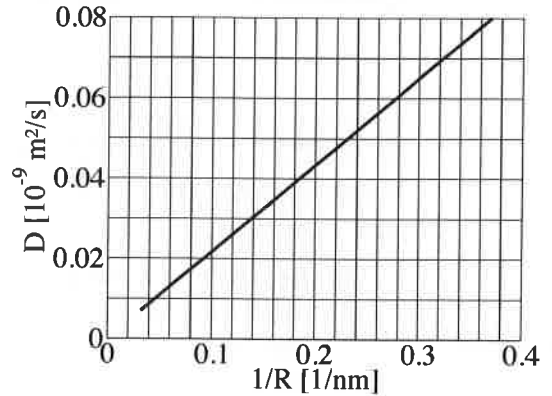
Keep in mind that the protein moves along the membrane.]

- b) How the time to diffuse to the other side of the cell would change if the cell radius would increase by a factor of 10, or by a factor of 100?

Assignment 3 [max. 6 points]

Using the Figure 1 on the right, calculate an estimate for the value of the Boltzmann coefficient. [Hint 1: yes, assume diffusion to be Brownian motion. Hint 2: yes, diffusion depends on friction (or viscosity), but how. Hint 3: yes, diffusion depends on temperature, and particle size, but how.]

Figure: Diffusion coefficient D as a function of $1/R$, where R is the radius of the molecule. Diffusion takes place in water, whose viscosity has a value of $1 \times 10^{-3} \text{ Ns} / \text{m}^2$. (Adapted from measurements of globular biological molecules in water at a temperature of 20 degrees Celsius.)



Assignment 4 [max. 5 points]

1. During the course we discussed how bacteria and fish swim, and how their swimming depends on the Reynolds number. Now, estimate how fast (or slowly) should you swim in water so that you would not have to care about inertia (such that friction would dominate how you move)? Keep again in mind that the (dynamic) viscosity of water is $1 \times 10^{-3} \text{ Ns} / \text{m}^2$, and now the temperature is 293 K.
2. Consider two imaginary polymers. Polymer A has a persistence length of 50 nm. Polymer B has a persistence length of 2 nm. Now imagine that you could grab the polymer at two points separated by 10 nm and bend it. Which polymer would be easier to bend and why? If you would use these two polymers as models for single- and double-stranded DNA, which one would be a better model for which? Why?

Assignment 5 [max. 4 points]

Consider the Debye (Debye-Huckel) screening length λ_D at the physiological temperature (310 K): $\lambda_D = (8\pi l_B c_\infty)^{-1/2}$. Here, $l_B = e^2 / (4\pi\epsilon\epsilon_0 k_B T)$ is the Bjerrum length, e is the elementary charge constant, ϵ is the dielectric constant, k_B is the Boltzmann constant, T is temperature, and c_∞ is the (bulk) salt concentration. We assume here monovalent salt in water (with relative permittivity around 80).

- Discuss how the Debye screening length depends on the salt concentration?
- What does the Bjerrum length describe?
- What is the screening length for monovalent salt at a concentration of 10 mM (the concentration of NaCl inside a mammalian cell) and 100 mM (the concentration of KCl inside a mammalian cell)? How about divalent salt at a concentration of 50 nM (nanomoles; that is the concentration of calcium inside a mammalian cell)?

- Then consider life inside a crowded cell. Assume water to occupy 70 volume percent of the volume, and 30 volume percent to be taken by macromolecular complexes (such as liposomes). Assume all of these macromolecular complexes to be identical, spherical, and to have exactly the same size (radius equal to 5 nm). Calculate a suggestive value for the distance between the macromolecular complexes (there are many ways to occupy the space by the macromolecular complexes, so consider one of them and just find an educated guess for the distance). Next, how does this distance compare to the Debye screening length you have computed above?

These might be useful:

Taylor expansion

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$$

Boltzmann constant $1.380662 \cdot 10^{-23} \text{ J / K}$

1 amu = $1.66 \times 10^{-27} \text{ kg}$ (atomic mass unit)

Avogadro's number $6.022 \cdot 10^{23} \text{ 1/mol}$

permittivity of vacuum $8.85 \cdot 10^{-12} \text{ F/m}$ (or $\text{coul}^2/\text{Nm}^2$)

permittivity of water ≈ 80 times that of a vacuum

charge on a proton $1.6 \cdot 10^{-19} \text{ coul}$

viscosity of water $\eta = 10^{-3} \text{ kg / ms}$

viscosity of syrup ≈ 3000 times that of water

mass density of syrup ≈ 1.4 times that of water

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\pi}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$\frac{dy}{dt} + Ay = B$, where A, B are constants, is solved by $y = y(t_0)e^{At_0}e^{-At} - \frac{B}{A}e^{At_0}e^{-At} + \frac{B}{A}$ ($t_0 = \text{initial time}$)

Elementary charge:

e is about $1.6021766208(98) \times 10^{-19} \text{ C}$

Reynold's number Re . This is one of the few equations/definitions that would be good to remember! But, if you don't, try to remember a little more about this. Re describes the ratio of inertia to friction, doesn't it. What are the dimensions of Re ? We remember from the lectures that the value of Re is affected by the flow speed, the size of the particle/object in the flow, and the friction of the liquid. But there are other things that affect it, too, right? Using dimensional analysis could be smart if you don't already memorize the exact equation.