FYS2010 Mathematical Methods of Physics Ia 16 June 2023

The students are allowed calculators.

1. Find all the complex solutions of the equation $\sin \pi z = 0$.

2. Show that the function $v(x, y) = e^{-y} \sin x$ is harmonic and find the holomorphic function $f(z) = u(x, y) + iv(x, y)$.

3. Calculate the integral

$$\oint_\gamma \frac{\cos z}{(z+i)^3} dz \,,$$

along the curve $\gamma(t) = -i + e^{it}$, $t \in [0, 2\pi]$.

4. Expand the function

$$f(z) = \frac{1}{1 - z^2}$$

in powers of z in the regions

$$\text{a)} \;\; 0 \leq |z| < 1 \,, \quad \text{b)} \;\; |z| > 1.$$

5. Calculate the following integral, using the theorem of residues:

$$\int_0^\infty \frac{dx}{x^4 + 1} \,.$$