GENERAL RELATIVITY I 4.3.2024 Exam, spring 2024

Write your student number on every sheet of paper.
DO NOT write your name on any of the papers.
It's not enough to do the calculations, you should also explain what you do.

1. Explain the following concepts/answer the following questions in a few lines. (Equations are not necessary, but if you use them, explain what the symbols are and what they mean.)

   a) How does special relativity unify spatial rotations and Galilei transformations?

   b) Explain the weak and strong equivalence principle **starting from general relativity**.

   c) If we integrate the components of a tensor over a proper volume element, why do the resulting quantities **not** form the components of a tensor? (Excluding the case of scalars.)

2. Consider 1+1-dimensional Minkowski space in Cartesian coordinates.

   a) Derive the Lorentz transformation matrix $\Lambda^\alpha{}_\beta$ from the condition that the transformation leaves the line element

   $$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2$$

   invariant. (You may neglect time reversal and spatial reflection.)

   b) Express the transformation parameter in terms of the velocity $v \equiv \frac{\mathrm{d}x}{\mathrm{d}t}$.

3. Consider the spatially flat FLRW universe with the metric

   $$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j \ .$$

   Calculate the components R_{0i0j} and R_{ijkl} of the Riemann tensor.

4. Consider the linearly perturbed Minkowski metric

   $$\mathrm{d}s^2 = -(1+2\phi)\mathrm{d}t^2 + (1-2\phi)\delta_{ij}\mathrm{d}x^i \mathrm{d}x^j \ .$$

   Show that in the weak field and slow motion limit, the geodesic equation for timelike geodesics reduces to the Newtonian equation of motion $\frac{\mathrm{d}^2 x^i}{\mathrm{d}t^2} = -\delta^{ij}\partial_j\phi$.

5. Show that in the Schwarzschild spacetime photons can have circular geodesics. Show that such orbits are unstable.