

You may answer in Finnish, Swedish, or English. Do problems 2 and 3 each on a different sheet of paper. Remember to put your name and student number on every separate sheet of paper. It is not enough to do calculations; an explanation about what is being done is needed also. Using a calculator is allowed. Exam time is 4 hours.

1. Calculate the age-redshift relation, the comoving-distance-redshift relation, the present age and the horizon distance (in units of H_0^{-1}) in the radiation-dominated universe ($\Omega_r = \Omega_0 = 1$).
2. Find $H(z)$ for a non-flat ($\Omega_k \neq 0$) universe, which contains matter, radiation, and dark energy (instead of vacuum energy), where the dark energy equation of state is given by

$$w(a) = w_0 + w_a(1 - a),$$

a is the scale factor normalized to 1 today, and w_0 and w_a are constants.

3. We say the universe is transparent when the photon mean free path λ_γ is larger than the Hubble length $l_H = H^{-1}$, and opaque when $\lambda_\gamma < l_H$. The photon mean free path is $\lambda_\gamma = 1/(\sigma_T n_e)$, where $n_e = x n_e^*$ is the number density of free electrons, n_e^* is the total number density of electrons, and x is the ionization fraction. The cross section for photon-electron scattering is $\sigma_T = \frac{8\pi}{3}(\alpha/m_e)^2$, where $\alpha = 1/137$ is the fine-structure constant. In recombination x falls from 1 to 10^{-4} . Assume an instantaneous recombination at $1 + z = 1300$. Show that the universe is opaque before recombination and transparent after recombination. Assume a matter-dominated universe with $\Omega_m = 1.0$ (note that Ω_m includes nonbaryonic matter), $\Omega_\Lambda = 0$, $h = 0.7$, and $\eta = 4 \times 10^{-10}$. You may ignore neutrons, i.e., assume that ordinary matter is all hydrogen.
4. Calculate how much ${}^4\text{He}$ is formed in Big Bang nucleosynthesis, using a simple model, where the neutrinos decouple instantaneously at $T = 0.9$ MeV, and the nucleosynthesis happens instantaneously at $T = 0.07$ MeV. Assume $\mu_n = \mu_p$ before neutrino decoupling. (You can use $g_* = 3.384$ for the whole time period).
5. Write an essay (max 500 words) on the events in the early universe during the temperature range $T = 10$ MeV \rightarrow 0.1 eV. No calculations or complicated equations expected.

Energy continuity equation:

$$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}$$