Basics of Monte Carlo simulations 2023

Final exam 10.03.2023

1. (**6p**) Describe the approaches realized in twisted generalized feedback shift register and Mersenne twister random number generators. Write down the algorithms. What is the purpose of the twists?
2. (**6p**) Describe the Markov chain Monte Carlo algorithm. Can it be used to generate random numbers? If yes, what kind? If not, why do you think so? Motivate your answer. Why is ergodicity of the function important for this method? What is the difference between Metropolis and Metropolis-Hastings method?
3. **(6p)** How to test random number generators? Describe shortly all the tests you remember. What test did you find the most reasonable? Why?
4. **(8p)** How to speed up the convergence of the result of an MC integration with increase of statistics faster than $1/\sqrt{N}$? Prove that the variance of MC estimation of integration of a function is decreasing when the partially stratified method is used.
5. (**6p**) Create a two-dimensional cellular automaton by describing the algorithm for your system to evolve. Write the algorithm in words and translate it into the rule number. Give the number of the rule in the general and totalistic notation. Why are they different?