Exam 02.11.2016

EVOLUTION AND THE THEORY OF GAMES

Please note the following:
- Duration of exam is 2,5 hours;
- Use of lecture notes NOT allowed;

Question 1.
In a 2-person game with strategy set $\mathbb{X}$ let $E(x, y)$ denote the payoff to strategy x against strategy y. Show that if x^* and y^* are both evolutionarily stable strategies(ESS), then the support of x^* is not a subset of the support of y^*.

Question 2.
Two rabbits are sitting side by side in a field. When a fox comes into the field, each rabbit can choose to run or to hide. If one rabbit runs while the other hides, the fox captures the runner with probability P, while the other one gets away quietly. If both rabbits run, then the fox chooses one at random, while the other escapes. If both rabbits keep hiding, the fox eventually finds them anyways, and then both will have to run. The fox chooses one at random, but the capture probability now is $Q > P$, because of the shorter distance. Nice story, but it doesn't really matter: this is a game between two rabbits, and the payoff matrix is given in terms of the escape probability:

	Run	Hide
Run	$1 - \frac{1}{2}P$	$1 - P$
Hide	1	$1 - \frac{1}{2}Q$

(payoffs to the row player)

Find all ESS-s (pure and mixed) depending on the values of P and Q, but always assuming that $0 < P < Q < 1$.

Question 3.
Same situation as in the previous question, but now one rabbit is faster than the other, and they both know it. The capture probability of the faster one is a fraction θ of the capture probability of the slower one. Never mind the story, if you don't like it. The point is that this is an asymmetric game with payoff matrix:

	Run	Hide
Run	$1 - \frac{1}{2}\theta P$, $1 - \frac{1}{2}P$	$1 - \theta P$, 1
Hide	1 , $1 - P$	$1 - \frac{1}{2}\theta Q$, $1 - \frac{1}{2}Q$

Find all ESS-s depending on the values of P, Q and θ, but always assuming that $0 < P < Q < 1$ and $\theta \in (0, 1)$.

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Question 4.
Same situation as in Question 2, but now we give each rabbit the choice how close it lets the fox approach before it starts running. Let $s \in [0,1]$ be the distance between rabbits and the fox. When the fox enters the field, the distance is $s = 1$. The capture probability $\varphi(s)$ is a continuous and decreasing function of the distance with $\varphi(0) = Q$ and $\varphi(1) = P$ and $0 < P < Q < 1$. For arbitrary pure strategies $s_1, s_2 \in [0,1]$ the payoff to s_1 against s_2 is

$$E(s_1, s_2) = \begin{cases} 1 & \text{if } \ s_1 < s_2 \\ 1 - \frac{1}{2}\varphi(s_1) & \text{if } \ s_1 = s_2 \\ 1 - \varphi(s_1) & \text{if } \ s_1 > s_2. \end{cases}$$

Show that if $P < \frac{1}{2}Q$, then there exists no pure strategy ESS and also not a mixed strategy ESS that can be represented by a probability density $f : [0,1] \to \mathbb{R}_+$.

Question 5.
Consider a two-stage Hawk-Dove game where after a H×H contest the players move to a different stage until the loser has recovered from its injuries while the winner gets the resource for free. The two stages of the game, Γ_1 and Γ_2, have the following payoff matrices:

Γ_1	Hawk	Dove
Hawk	$\frac{1}{2}(R + \delta\,\Gamma_2^{\text{row}}) + \frac{1}{2}\delta\,\Gamma_2^{\text{col}}$	$R + \delta\,\Gamma_1$
Dove	$\delta\,\Gamma_1$	$\frac{1}{2}R + \delta\,\Gamma_1$

(payoffs to the row player)

and

Γ_2	Loser
Winner	$R + (1-\varepsilon)\,\delta\,\Gamma_1 + \varepsilon\,\delta\,\Gamma_2^{\text{row}}\ ,\ \ (1-\varepsilon)\,\delta\,\Gamma_1 + \varepsilon\,\delta\,\Gamma_2^{\text{col}}$

(asymmetric game)

where $R > 0$ is the value of the resource, $\delta \in (0,1)$ the stage-independent probability of a next round, and $\varepsilon \in (0,1)$ the probability of recovery in the next round. What is the expected number of rounds of a single play on stage Γ_2 before returning to stage Γ_1 again? Calculate the payoff matrix for the full game $\Gamma = (\Gamma_1, \Gamma_2)$ and find all ESS-s (pure and mixed).